Hong Kong Mathematics Olympiad (2012 / 2013)

Final Event 1 (Group)

香港数学竞赛 (2012 / 2013)

决赛项目 1 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 求 $(2^{13}+1)(2^{14}+1)(2^{15}+1)(2^{16}+1)$ 的个位数字。

Find the unit digit of $(2^{13}+1)(2^{14}+1)(2^{15}+1)(2^{16}+1)$.

2. 求 $16\div(0.40+0.41+0.42+...+0.59)$ 的值的整数部分。

Find the integer part of $16\div(0.40+0.41+0.42+...+0.59)$.

3. 从 1、2、4、6、7 中选三个数字组成三位数。这些三位数有多少个能被 3 整除？

Choose three digits from 1, 2, 4, 6, 7 to construct three-digit numbers. Of these three-digit numbers how many of them are divisible by 3?

4. 用 1、2、3、4、5、6 组成一个 6 位数：*ABCDEF*，使得：*A* 能被 1 整除，*AB* 能被 2 整除，*ABC* 能被 3 整除，*ABCD* 能被 4 整除，*ABCDE* 能被 5 整除，及 *ABCDEF* 能被 6 整除。求 *A* 的最大值。

Using numbers: 1, 2, 3, 4, 5, 6 to form a six-digit number: *ABCDEF* such that *A* is divisible by 1, *AB* is divisible by 2, *ABC* is divisible by 3, *ABCD* is divisible by 4, *ABCDE* is divisible by 5 and *ABCDEF* is divisible by 6. Find the greatest value of *A*.

Hong Kong Mathematics Olympiad (2012 / 2013)

Final Event 2 (Group)

香港数学竞赛 (2012 / 2013)

决赛项目 2 (团体)

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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 $4^3+4^r+4^4$ 是一平方数，其中 r 是正整数，求 r 的最小值。

If $4^3+4^r+4^4$ is a perfect square and r is a positive integer, find the minimum value of r.

2. 三男 B_1、B_2、B_3 和三女 G_1、G_2、G_3 就坐一排座位，并满足以下两个条件:
1) 一男不会坐在另一男旁边及一女不会坐在另一女旁边
2) B_1 必须坐在 G_1 旁边
若 s 是这样就坐的排列数量，求 s 的值。

Three boys B_1, B_2, B_3 and three girls G_1, G_2, G_3 are to be seated in a row according to the following rules:
1) A boy will not sit next to another boy and a girl will not sit next to another girl
2) Boy B_1 must sit next to girl G_1
If s is the number of different such seating arrangements, find the value of s.

3. 设 $f(x)=\dfrac{x+a}{x^2+\frac{1}{2}}$，$x$ 为实数且 $f(x)$ 的最大值和最小值分别是 $\frac{1}{2}$ 和 -1。若 $t=f(0)$，求 t 的值。

Let $f(x)=\dfrac{x+a}{x^2+\frac{1}{2}}$, where x is a real number and the maximum value of $f(x)$ is $\frac{1}{2}$ and the minimum value of $f(x)$ is -1. If $t=f(0)$, find the value of t.

4. 如图一，ABC 是一等腰三角形，其中$\angle ABC = u^\circ$，$AB = BC = a$ 和 $AC = b$。若二次方程

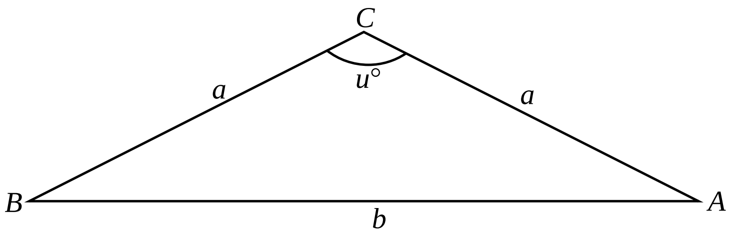
$$ax^2 - \sqrt{2} \cdot bx + a = 0$$

有两个实根，它们的绝对差为$\sqrt{2}$，求u的值。

In Figure 1, ABC is an isosceles triangle with $\angle ABC = u^\circ$, $AB = BC = a$ and $AC = b$.
If the quadratic equation

$$ax^2 - \sqrt{2} \cdot bx + a = 0$$

has two real roots, whose absolute difference is $\sqrt{2}$, find the value of u.



图一

Figure 1

Hong Kong Mathematics Olympiad (2012 / 2013)

Final Event 3 (Group)

香港数学竞赛 (2012 / 2013)

决赛项目 3 (团体)

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1. 若 m 和 n 是正整数且 $m^2 - n^2 = 43$，求 $m^3 - n^3$ 的值。

If m and n are positive integers with $m^2 - n^2 = 43$, find the value of $m^3 - n^3$.

2. 设 x_1、x_2、…、x_{10} 为非零整数，且满足 $-1 \le x_i \le 2$ 其中 i = 1、2、…、10。若 $x_1 + x_2 + ... + x_{10} = 11$，求 $x_1{}^2 + x_2{}^2 + ... + x_{10}{}^2$ 的最大可能值。

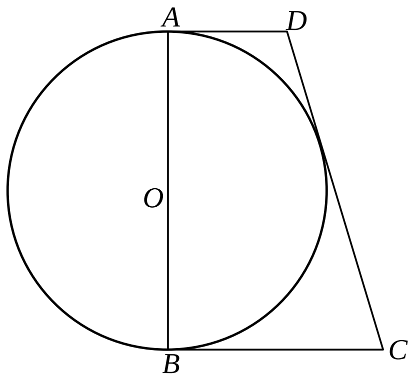
Let $x_1, x_2, \ldots, x_{10}$ be non-zero integers satisfying $-1 \le x_i \le 2$ for $i = 1, 2, \ldots, 10$. If $x_1 + x_2 + \ldots + x_{10} = 11$, find the maximum possible value for $x_1^2 + x_2^2 + \ldots + x_{10}^2$.

3. 若 $f(n) = a^n + b^n$，其中 n 是正整数且 $f(3) = [f(1)]^3 + f(1)$，求 $a \cdot b$ 的值。

If $f(n) = a^n + b^n$, where n is a positive integer and $f(3) = \left[f(1)\right]^3 + f(1)$, find the value of $a \cdot b$.

4. 如图二，AD，BC 和 CD 是以 O 作圆心且直径 $AB=12$ 的圆的切线。若 $AD=4$，求 BC 的值。

In Figure 2, AD, BC and CD are tangents to the circle with centre at O and diameter $AB=12$. If $AD=4$, find the value of BC.



图二

Figure 2

Hong Kong Mathematics Olympiad (2012 / 2013)
Final Event 4 (Group)
香港数学竞赛 (2012 / 2013)
决赛项目 4 (团体)

除非特别声明，答案须用数字表达，并化至最简。
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1. 若 P 为整数 3,659,893,456,789,325,678 与 342,973,489,379,256 的乘积，求 P 的位数。

If P be the product of 3,659,893,456,789,325,678 and 342,973,489,379,256, find the number of digits of P.

2. 若 $\frac{1}{4}+4\left(\frac{1}{2013}+\frac{1}{x}\right)=\frac{7}{4}$，求 $1872+48\times\left(\frac{2013x}{x+2013}\right)$ 的值。

If $\frac{1}{4}+4\left(\frac{1}{2013}+\frac{1}{x}\right)=\frac{7}{4}$, find the value of $1872+48\times\left(\frac{2013x}{x+2013}\right)$.

3. 有一个正整数被 10 除，余数为 9；被 9 除，余数为 8；被 8 除，余数为 7；等等直至被 2 除，余数为 1。求此正整数的最小值。

The remainders of a positive integer when divided by 10, 9, 8, …, 2 are 9, 8, 7, …, 1, respectively. Find the smallest such positive integer.

4. 如图三，A、B、C、D、E 代表不同的个位数字。求 $A+B+C+D+E$ 的值。

In Figure 3, A, B, C, D, E represent different digits. Find the value of $A+B+C+D+E$.

$$\begin{array}{r} ABCDE \\ \times \quad\quad 9 \\ \hline 1AAA0E \end{array}$$

图三

Figure 3